SOLVING QUADRATIC EQUATIONS

A Jenkins-Bryant production

Solving using the quadratic equation

A **quadratic equation** is always written in the form of:

$$ax^2 + bx + c = 0$$

Solution Using the Quadratic Formula

Factoring is useful only for those quadratic equations which have whole numbers. When you encounter quadratic equations that can not be easily factored out, use the quadratic formula to find the value of x:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Steps

- Step 1 : Rewrite the equation if it is not in standard form
- Step 2 : Identify the a, b, c
- Step 3 : put the numbers in the equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Step 4 once you have solved if you are able to simplify your solution

Examples:

$$x^2 - 8 = -2x$$

$$x^2 + 2x - 8 = 0 \quad \leftarrow \text{Rewrite in standard form, where } a = 1,\ b = 2, \text{ and } c = -8$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-8)}}{2(1)} \quad \leftarrow \text{Plug in numbers into the equation}$$

$$= \frac{-2 \pm \sqrt{36}}{2(1)}$$

$$= \frac{-2 \pm 6}{2}$$

$$= 2, -4 \quad \leftarrow \text{The two rational solutions}$$

Example

Let's Do One

1. $x^2 - 2x + 1 = 0$

Vocabulary

binomial expression is an algebraic expression consisting of two terms or monomials separated by a plus (+) or minus (−) sign.

complex conjugate

The complex conjugate of any complex number $a + bi$, denoted $\overline{a + bi}$, is $a - bi$.

complex number

Any number that can be written as $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$.

expression

A mathematical phrase that contains operations, numbers, and/or variables.

nth root it is the "radical" symbol (used for square roots) with a little n to mean nth root.

standard form of a polynomial
A polynomial in one variable is written in standard form when the terms are in order from greatest degree to least degree.

rational expression
An algebraic expression whose numerator and denominator are polynomials and whose denominator has a degree ≥ 1.

VOCABULARY

rational number
A number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

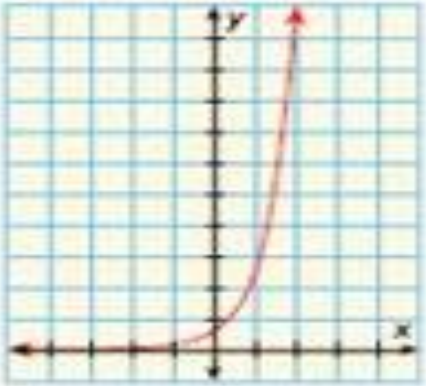
rational exponent
An exponent that can be expressed as $\frac{m}{n}$ such that if m and n are integers, then $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$.

Vocabulary

trinomial
A polynomial with three terms.

whole number
The set of natural numbers and zero.

properties of operations as strategies to multiply and divide

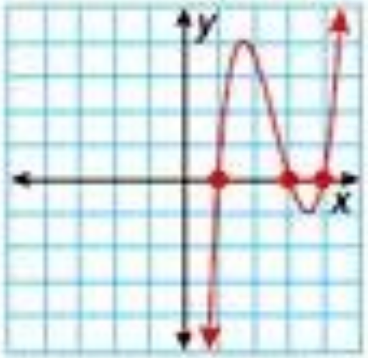


$f(x) = 3 \cdot 4^x$

exponential function

A function of the form $f(x) = ab^x$, where a and b are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.

Vocabulary



$f(x) = x^3 - 8x^2 + 19x - 12$

polynomial function

A function whose rule is a polynomial.

Practice

2. $x^2 + 9x + 20 = 0$

3. $3x^2 - 5x - 12 = 0$

COMPLETING THE SQUARE

Steps

Example: Solve $x^2 + 6x + 5 = 0$ by completing the square.

1) If the leading coefficient is not 1, use the multiplication (or division) property of equality to make it 1:

$$x^2 + 6x + 5 = 0 \quad \leftarrow \text{In this case the leading coefficient is already 1}$$

2) Rewrite the equation by sending the constant to the right side of the equation:

$$x^2 + 6x + 5 = 0$$
$$x^2 + 6x + 5 - 5 = 0 - 5$$
$$x^2 + 6x = -5$$

Steps

3) Divide the numerical coefficient the middle term by 2, then square it, and add it to both sides of the equation, but leave the square form on the left side of the equation:

$$x^2 + 6x = -5$$

$$x^2 + 6x + (3)^2 = -5 + (3)^2$$

$$x^2 + 6x + (3)^2 = -5 + 9$$

$$x^2 + 6x + (3)^2 = 4$$

Middle term coefficient = 6 $\qquad \frac{6}{2} = 3 \rightarrow (3)^2$

4) Once you found the squared number rewrite the equation as follows:

$$x^2 + 6x + (3)^2 = 4$$

$$(x + 3)^2 = 4$$

← Bring down the variable x and put it inside the parentheses

← Use the sign of the middle term. In this case it is +.

← Write the squared number. In this case it is 3.

The resultant binomial is $(x+3)^2 = 4$

Steps

5) Using the square root property clear the term.

$$\sqrt{(x+3)^2} = \pm\sqrt{4}$$ ← The square root of a squared term is the term by itself.

$$x+3=\pm 2$$

6) Solve for the variable x.

$$x+3=\pm 2$$ ← The $\pm$ notation is used because the square root can have both positive and negative answers.

$x+3=2$	$x+3=-2$
$x=2-3$	$x=-2-3$
$x=-1$	$x=-5$

$(-1+3)^2=(2)^2=4$ And $(-5+3)^2=(-2)^2=4$

Checking

7) Check your solution.

$x = -1$

$x^2 + 6x + 5 = 0$

$(-1)^2 + 6(-1) + 5 = 0$

$1 - 6 + 5 = 0$

$0 = 0$

← Both solutions are true:

$x = -5$

$x^2 + 6x + 5 = 0$

$(-5)^2 + 6(-5) + 5 = 0$

$25 - 30 + 5 = 0$

$0 = 0$

COMPUTING EXPONETS